NAG Fortran Library Routine Document

F08UFF (SPBSTF/DPBSTF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08UFF (SPBSTF/DPBSTF) computes a split Cholesky factorization of a real symmetric positive-definite band matrix.

2 Specification

```
SUBROUTINE F08UFF(UPLO, N, KB, BB, LDBB, INFO)
ENTRY      spbstf (UPLO, N, KB, BB, LDBB, INFO)
INTEGER           N, KB, LDBB, INFO
real              BB(LDBB,*)
CHARACTER*1       UPLO
```

The ENTRY statement enables the routine to be called by its LAPACK name.

3 Description

This routine computes a split Cholesky factorization of a real symmetric positive-definite band matrix B. It is designed to be used in conjunction with F08UEF (SSBGST/DSBGST).

The factorization has the form $B = S^T S$, where S is a band matrix of the same bandwidth as B and the following structure: S is upper triangular in the first $(n+k)/2$ rows, and transposed — hence, lower triangular — in the remaining rows. For example, if $n = 9$ and $k = 2$, then

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & & & & & & \\ & s_{22} & s_{23} & s_{24} & & & & & \\ & & s_{33} & s_{34} & s_{35} & & & & \\ & & & s_{44} & s_{45} & & & & \\ & & & & s_{55} & & & & \\ & & & s_{64} & s_{65} & s_{66} & & & \\ & & & & s_{75} & s_{76} & s_{77} & & \\ & & & & & s_{86} & s_{87} & s_{88} & \\ & & & & & & s_{97} & s_{98} & s_{99} \end{pmatrix}.$$

4 References

None.

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: indicates whether the upper or lower triangular part of B is stored as follows:

if UPLO = 'U', the upper triangular part of B is stored;

if UPLO = 'L', the lower triangular part of B is stored.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix B.

Constraint: $N \geq 0$.

3: KB – INTEGER *Input*

On entry: k, the number of super-diagonals of the matrix B if UPLO = 'U', or the number of sub-diagonals if UPLO = 'L'.

Constraint: $KB \geq 0$.

4: BB(LDBB,*) – ***real*** array *Input/Output*

Note: the second dimension of the array BB must be at least $\max(1, N)$.

On entry: the n by n symmetric positive-definite band matrix B, stored in rows 1 to $k+1$. More precisely, if UPLO = 'U', the elements of the upper triangle of B within the band must be stored with element b_{ij} in $BB(k+1+i-j, j)$ for $\max(1, j-k) \leq i \leq j$; if UPLO = 'L', the elements of the lower triangle of B within the band must be stored with element b_{ij} in $BB(1+i-j, j)$ for $j \leq i \leq \min(n, j+k)$.

On exit: B is overwritten by the elements of its split Cholesky factor S.

5: LDBB – INTEGER *Input*

On entry: the first dimension of the array BB as declared in the (sub)program from which F08UFF (SPBSTF/DPBSTF) is called.

Constraint: $LDBB \geq KB + 1$.

6: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, the factorization could not be completed, because the updated element b_{ii} would be the square root of a negative number. Hence B is not positive-definite. This may indicate an error in forming the matrix B.

7 Accuracy

The computed factor S is the exact factor of a perturbed matrix $B+E$, where

$$|E| \leq c(k+1)\varepsilon|S^T|\,|S|,$$

$c(k+1)$ is a modest linear function of $k+1$, and ε is the ***machine precision***. It follows that $|e_{ij}| \leq c(k+1)\varepsilon\sqrt{(b_{ii}b_{jj})}$.

8 Further Comments

The total number of floating-point operations is approximately $n(k+1)^2$, assuming $n \gg k$.

A call to this routine may be followed by a call to F08UEF (SSBGST/DSBGST) to solve the generalized eigenproblem $Az = \lambda Bz$, where A and B are banded and B is positive-definite.

The complex analogue of this routine is F08UTF (CPBSTF/ZPBSTF).

9 Example

See Section 9 of the document for F08UEF (SSBGST/DSBGST).